JOURNAL OF COMPUTER AND SYSTEM SCIENCES 31, 343–354 (1985)

Interactions between Dependencies and Nested Relational Structures*

PATRICK C. FISCHER

Department of Computer Science, Vanderbilt University, P. O. Box 1679, Station B, Nashville, Tennessee 37235

LAWRENCE V. SAXTON

Department of Computer Science, University of Regina, Regina, Saskatchewan, Canada S4S 0A2

STAN J. THOMAS

Department of Mathematics and Computer Science, Wake Forest University, Winston-Salem, North Carolina 27109

AND

DIRK VAN GUCHT

Department of Computer Science, Vanderbilt University, P. O. Box 1679, Station B, Nashville, Tennessee 37235

Received December 17, 1984; revised March 22, 1985

Nesting is a way of transforming a first-normal-form relation into a structure with set-valued entries in some positions instead of atomic entries. In this paper we study how functional and multivalued dependencies interact with nesting. We describe how nesting preserves, alters, or destroys dependencies holding in a first-normal-form relation. We then consider dependencies which hold in each block of the horizontally decomposed relation induced by nesting and study the relationship between these "local" dependencies and "global" dependencies in the normalized relation.

1. INTRODUCTION

The relational database model has been the subject of a great deal of research during the past decade since it was introduced by Codd [10]. Since this fundamental paper was published, the principal research models have been the normal forms of the relational model. Recently, however, the integration of allied functions such

* This research was supported in part by National Science Foundation Grant MCS-8115106 and a Natural Sciences and Engineering Research Council of Canada Grant A0334.

0022-0000/85 $3.00

as information retrieval of textual data, image processing, and form processing has been proposed [2, 4, 17, 19, 21]. For these applications, the standard first normal form (1NF) relational model must be enhanced to handle less structured data, such as hierarchies and repeating values. The idea of relaxing the first normal form restriction was put forward by Makinouchi in 1977 [20]. Jaeschke and Schek [18] studied two operators, nesting, and unnesting, to allow such a relaxation. Fischer and Thomas [14, 23, 24] analyzed these operators in a general setting and studied their interactions with the usual operators of the relational algebra.

In this research, we study the interaction of the nest and unnest operators with functional and multivalued dependencies. Dependencies were incorporated in the relational database model to represent integrity constraints in the system being modeled and have proven useful in the scheme design process. Functional dependencies (FDs) were introduced by Codd [11] and further characterized in [1]. The role of FDs in normalization theory was developed in [9, 8, 6, 5, 3]. Delobel [12], Fagin [13] and Zaniolo [25] independently described multivalued dependencies (MVDs) to deal with relationships among set values.

We will begin by considering the concepts of nesting and unnesting. We next generalize the notion of satisfaction of FDs and MVDs to deal with non-1NF relational structures. We study the relationship of dependencies which hold in the unnested (1NF) vs the nested form of a relation. Finally, we consider dependencies which hold in each block of the horizontally decomposed relation instance induced by nesting [16, 22]. We study the relationship between those "local" dependencies and "global" dependencies in the unnested relation.

2. Basic Definitions

We will be using standard notation for relational databases. An attribute is a symbol taken from a finite set, called a universe, $U = \{A_1, A_2, ..., A_n\}$. For each attribute A, there is a set of possible values called its *domain*, denoted $\mathrm{DOM}(A)$. A *tuple* t over U is a mapping from U into the union of the attribute domains such that for each $A \in U$, $t(A) \in \mathrm{DOM}(A)$. The *X-value* of a tuple t, denoted $t[X]$, where $X \subset U$, is the restriction of this mapping to the set X. We will use capital letters from the beginning of the alphabet for single attributes and capital letters from the end of the alphabet for sets of attributes. The notation XY will be used to represent $X \cup Y$, for X, $Y \subset U$. Similarly, for $A \in U$, XA will mean $X \cup \{A\}$.

A *relation* r over U is a finite set of tuples over U. The *projection* of a relation r on X is the set $\{t[X] \mid t \in r\}$ and is denoted $\Pi_X(r)$.

The operators NEST and UNNEST were used by Jaeschke and Schek to convert normalized relations into unnormalized relations [18]. Informally, nesting on a set of attributes Z collects together into a set all tuples which agree on $U - Z$.

Let r be a relation over U and let Z be a nonempty subset of U. We define $r^* = \mathrm{NEST}_Z(r)$ as the relation over $(U - Z)Z^*$, where $\mathrm{DOM}(Z^*)$ is the set of relations over Z. A tuple t^* occurs in r^* if and only if there exists a tuple t in r such that:

(1) $t^*[U-Z]=t[U-Z]$ and

(2) $t^*(Z^*)=\Pi_Z(\{t' \in r \mid t'[U-Z]=t[U-Z]\})$.

The operator UNNEST is defined as the left inverse of NEST. Thus for a relation s^* with scheme V, where $Z^* \in V$, $\text{UNNEST}_{Z^*}(s^*)$ will produce the set of all tuples t over $(V-Z^*)Z$ such that for some tuple $t^* \in s^*$, $t[V-Z^*]=t^*[V-Z^*]$, and $t[Z] \in t^*(Z^*)$.

In [23, 24] it was shown that any valid sequence of NEST and UNNEST operations transforms a 1NF relation r into a structure which can be subsequently unnested to obtain r again. Thus, information representable in 1NF is not lost by nesting. On the other hand, nested structures may carry additional information which can be lost by unnesting [18, 23]. In order for our results to hold for an arbitrary structure r, we shall agree not to unnest any of the already nested attributes of r, i.e., to treat them as basic attributes with complex domains.

We extend the notions of satisfaction of FDs and MVDs in the natural way, with set-valued objects equal if and only if they are equal as sets. Thus, a relation r (not necessarily in 1NF) satisfies the *functional dependency* $X \rightarrow Y$ if and only if for all tuples t_1, $t_2 \in r$, whenever $t_1[X]=t_2[X]$, $t_1[Y]=t_2[Y]$. Further, a relation r satisfies the *multivalued dependency* $X \twoheadrightarrow Y$ if and only if for all tuples t_1, $t_2 \in r$, if $t_1[X]=t_2[X]$ there exists a tuple $t_3 \in r$ with $t_3[U-XY]=t_1[U-XY]$ and $t_3[XY]=t_2[XY]$. We may also write $X \twoheadrightarrow Y \mid (U-XY)$ to emphasize the complementation property of MVDs [7].

We must also note that FDs and MVDs might be valid in some subset of a relation but not in the entire relation. Such a dependency might be considered *local* to this subrelation. The standard definition requires a dependency to hold *globally*. We will first study global dependencies.

3. Nesting and Functional Dependencies

We now investigate the interaction between nesting and FDs. In each case, we shall consider two structures r and r^* which satisfy $r^*=\text{NEST}_Z(r)$. IIf U denotes the scheme of r with $Z \subset U$, then the scheme of r^* will be $U^*=(U-Z)Z^*$.

Lemma 1. *The* FD $(U-Z) \rightarrow Z^*$ *holds in* r^*.

Proof. Immediate, from the definition of nesting (cf. [18, 23]). ∎

We first assume that nesting does not involve the left-hand side of an FD. We than vary the relationship of the nesting set Z to $U-XY$, the set of attributes not involved in the FD. In the first subcase, Z is contained in $U-XY$.

THEOREM 1. *Assume* $XY \cap Z = \varnothing$. *Then the* FD $X \to Y$ *holds in* r *if and only if* $X \to Y$ *holds in* r^*.

Proof. Immediate, since $\Pi_{XY}(r) = \Pi_{XY}(r^*)$. ∎

We now consider the subcase where Z is disjoint from $U - XY$. We first prove:

LEMMA 2. *Assume* $X \cap Z = \varnothing$. *Then the* FD $X \to Z$ *holds in* r *if and only if*:

(a) $X \to Z^*$ *holds in* r^* *and*

(b) *for each tuple* $t^* \in r^*$, $t^*(Z^*)$ *is a singleton set.*

Proof. Let $W = U - XZ$. Then the scheme of r may be written XZW, and the scheme of r^* is XZ^*W. Assume $X \to Z$ holds in r. Suppose a set S in the Z^*-column of r^* is not a singleton. Then there exists a tuple of r^* of the form $\langle x, S, w \rangle$ and there exist two distinct members of S, say z_1 and z_2. But $\langle x, S, w \rangle$ must have resulted from at least two tuples of r, including $\langle x, z_1, w \rangle$ and $\langle x, z_2, w \rangle$, thus violating $X \to Z$ in r. Hence, the sets must be singletons. This fact yields a natural isomorphism between r and r^*, namely tuples with singleton sets in the Z^*-column of r^* correspond to tuples with a single data item in the Z-field of r and with corresponding values in all other columns. Under this condition $X \to Z$ holds in r if and only if $X \to Z^*$ holds in r^*. This completes the only if part. The converse follows immediately since condition (b) will guarantee the existence of the same isomorphism. ∎

THEOREM 2. *Assume* $X \cap Y = \varnothing$ *and* $Z \subset Y$. *Then* $X \to Y$ *holds in* r *if and only if*:

(a) $X \to (Y - Z)\, Z^*$ *holds in* r^* *and*

(b) *for each tuple* $t^* \in r^*$, $t^*(Z^*)$ *is a singleton set.*

Proof. From the decomposition property of FDs, we know $X \to Y$ holds in r if and only if $X \to Y - Z$ and $X \to Z$ both hold in r. Similarly, $X \to (Y - Z)\, Z^*$ holds in r^* if and only if $X \to (Y - Z)$ and $X \to Z^*$ both hold in r^*. The result then follows from Theorem 1 and Lemma 2. ∎

The next subcase is where Z contains $U - XY$.

THEOREM 3. *Assume* $X \cap Y = \varnothing$ *and* $Z \supset (U - XY)$. *Then* $X \to Y$ *holds in* r *if and only if*:

(a) $X \to (Y - Z)$ *holds in* r^* *and*

(b) $\Pi_{Y \cap Z}(t^*(Z^*))$ *is a singleton for each* $t^* \in r^*$.

Proof. If $X \to Y$ holds in r, then $X \to Y - Z$ also holds in r by decomposition. Then part (a) follows from Theorem 1. Part (b) is proved by an argument similar to that in Theorem 2. Now assume $X \to Y$ is violated in r. Then there exist tuples t_1, $t_2 \in r$ such that $t_1[X] = t_2[X]$ but either $t_1[Y - Z] \neq t_2[Y - Z]$ or $t_1[Y \cap Z] \neq$

$t_2[Y \cap Z]$. In the first case, $X \to Y - Z$ is violated in r^* since t_1 and t_2 will contribute to distinct tuples of r^* after nesting. In the second case, we have $t_1[XY - Z] = t_2[XY - Z]$. Thus, t_1 and t_2 contribute to the same tuple $t^* \in r^*$ and condition (b) must be violated. ▮

COROLLARY 1. *Under the assumptions of Theorem* 3, *if* $X \to Y$ *holds in* r, *then* $X \to Z^*$ *holds in* r^*.

Proof. From the assumptions, $X \subset (U - Z) \subset XY$. If $X \to Y$ holds in r then $X \to (U - Z)$ holds in r, hence in r^* by Theorem 1. From Lemma 1, $(U - Z) \to Z^*$ holds in r^*. The result follows from the transitivity property of FDs. ▮

The converse of Corollary 1 clearly does not hold.

As previously mentioned, we have been assuming $X \cap Z = \varnothing$. The final subcase is where Z splits $U - XY$ (i.e., $(U - XY) \cap Z \neq \varnothing$ and $(U - XY) - Z \neq \varnothing$). We feel very little of value can be inferred in this situation about global FDs in r vs r^* since if Z splits $U - XY$, one can easily satisfy FDs in r^* but violate their "natural" counterparts in r, and vice versa.

We now present two results dealing with nesting on the left hand side of an FD.

THEOREM 4. *Assume* $X \cap Y = \varnothing$ *and* $Z \subset X$. *Then the* FD $X \to Y$ *holds in* r *if and only if whenever there exist tuples* t_1, $t_2 \in r^*$ *such that* $t_1[X - Z] = t_2[X - Z]$ *and* $t_1(Z^*) \cap t_2(Z^*) \neq \varnothing$ then $t_1[Y] = t_2[Y]$.

Proof. Let $W = U - XY$. Then the scheme of r may be written $(X - Z)\, ZYW$ and the scheme of r^* will be $(X - Z)\, Z^*YW$. Assume $X \to Y$ holds in r. Suppose there are tuples $\langle u, S, y, w \rangle$ and $\langle u, S', y', w' \rangle$ in r^* with $S \cap S' \neq \varnothing$ and $y \neq y'$. Then for any $z \in S \cap S'$ the tuples $\langle u, z, y, w \rangle$ and $\langle u, z, y', w' \rangle$ must be in r, which would violate $X \to Y$. Now suppose $X \to Y$ is violated in r. Then there are tuples $\langle u, z, y, w \rangle$ and $\langle u, z, y', w' \rangle$ in r with $y \neq y'$. After nesting there will be tuples $\langle u, S, y, w \rangle$ and $\langle u, S', y', w' \rangle$ in r^* with $S \cap S' \neq \varnothing$, thus violating the condition. ▮

COROLLARY 2. *Under the assumptions of Theorem* 4, *if the* FD $X \to Y$ *holds in* r *then the* FD $(X - Z)\, Z^* \to Y$ *holds in* r^*.

Proof. Suppose t_1, $t_2 \in r^*$ satisfy the left hand side, i.e., $t_1[X - Z] = t_2[X - Z]$ and $t_1(Z^*) = t_2(Z^*)$. In particular, $t_1(Z^*) \cap t_2(Z^*) \neq \varnothing$. Then Theorem 4 implies $t_1[Y] = t_2[Y]$. ▮

Remark 1. The converse of Corollary 2 does not hold. A counterexample is given in Table I. The relation $r^* = \mathrm{NEST}_Z(r)$ satisfies $(X - Z)\, Z^* \to Y$ but r does not satisfy $(X - Z)\, Z \to Y$.

TABLE I

r				r^*			
$X-Z$	Z	Y	W	$X-Z$	Z^*	Y	W
u	z_1	y_1	w	u	$\{z\}$	y_1	w
u	z_1	y_2	w	u	$\{z_1, z_2\}$	y_2	w
u	z_2	y_2	w				

4. Nesting and Multivalued Dependencies

In the case of multivalued dependencies we will assume that nesting does not involve the left-hand side X of an MVD. Unlike the situation for FDs we will not be able to relax this assumpton later (see Remark 3 below). Because of the symmetry between the right-hand side Y of an MVD and the complement $U-XY$ we find two basic subcases: (1) the nesting set Z is contained in Y or $U-XY$ and (2) $Z=Y$ or $Z=U-XY$.

The first theorem on nesting and MVDs is straightforward. While it may be regarded as nesting "away" from the MVD, it is really an instance of the first subcase. What is surprising is the variety of interesting results derivable from it.

Theorem 5. *Assume $XY \cap Z = \emptyset$. Then the* MVD $X \twoheadrightarrow Y$ *holds in r if and only if $X \twoheadrightarrow Y$ holds in r^*.*

Proof. Let $W=U-XYZ$. Assume $X \twoheadrightarrow Y$ holds in r. We need to show that for any two tuples in r^* of the form $\langle x, y, S, w\rangle$ and $\langle x, y', S', w'\rangle$, r^* also contains the tuple $\langle x, y', S, w\rangle$. For each $z \in S$ and $z' \in S'$ we know from the definition of nesting, that $\langle x, y, z, w\rangle$ and $\langle x, y', z', w'\rangle$ are tuples of r. Since $X \twoheadrightarrow Y$ holds in r, we also have $\langle x, y', z, w\rangle \in r$. From this we can conclude that r^* contains a tuple $\langle x, y', \bar{S}, w\rangle$ with $\bar{S} \supset S$. Thus, we need only show that $\bar{S} \subset S$. For $\bar{z} \in \bar{S}$ and $z \in S$ we know that $\langle x, y', \bar{z}, w\rangle$ and $\langle x, y, z, w\rangle$ are tuples of r. Since $X \twoheadrightarrow Y$ holds in r, we also have $\langle x, y, \bar{z}, w\rangle \in r$. Therefore, $\bar{z} \in S$ and $\bar{S} = S$. Hence, $X \twoheadrightarrow Y$ holds in r^*.

To prove the convers we assume $X \twoheadrightarrow Y$ holds in r^* and must show that for any tuples in r of the form $\langle x, y, z, w\rangle$ and $\langle x', y', z', w'\rangle$, r also contains the tuple $\langle x, y', z, w\rangle$. By the definition of nesting on Z we know that r^* contains tuples $\langle x, y, S, w\rangle$ and $\langle x, y', S', w'\rangle$ such that $z \in S$ and $z' \in S'$, respectively. Since $X \twoheadrightarrow Y$ holds in r^*, we have $\langle x, y', S, w\rangle \in r^*$. Then unnesting yields $\langle x, y', z, w\rangle \in r$. ∎

Corollary 3. *Assume $X \cap Y = \emptyset$ and $Z \subset Y$. Then the* MVD $X \twoheadrightarrow Y$ *holds in r if and only if $X \twoheadrightarrow (Y-Z)\, Z^*$ holds in r^*.*

Proof. Let $W = U - XY$. Then the scheme of r is XYW and the scheme of r^* is $X(Y-Z)\,Z^*W$. From the complementation rule of MVDs we know that $X \twoheadrightarrow Y$ holds in r if and only if $X \twoheadrightarrow W$ holds in r. Since $XW \cap Z = \varnothing$, we may apply Theorem 5 to conclude that $X \twoheadrightarrow W$ holds in r if and only if $X \twoheadrightarrow W$ holds in r^*. By complementation in the scheme of r^* we obtain $X \twoheadrightarrow W$ holds in r^* if and only if $X \twoheadrightarrow (Y-Z)\,Z^*$ holds in r^*. This establishes the desired result. ▮

The next two theorems deal with the second subcase and show an important connection between MVDs and nested structures.

THEOREM 6. *Assume $X \cap Z = \varnothing$. Then the following statements are equivalent*:

(1) *$X \twoheadrightarrow Z$ holds in r.*

(2) *$X \twoheadrightarrow Z^*$ holds in $r^* = \mathrm{NEST}_Z(r)$.*

(3) *$X \rightarrow Z^*$ holds in $r^* = \mathrm{NEST}_Z(r)$.*

Proof. The equivalence of (1) and (2) follows immediately from Corollary 3 by setting $Z = Y$. (3) implies (2) is trivial since an FD logically implies the coresponding MVD. Now assume $X \twoheadrightarrow Z^*$ holds in r^*. From Lemma 1, $(U-Z) \rightarrow Z^*$ holds in r^*. Then $X \rightarrow Z^*$ follows from the second mixed rule for FDs and MVDs [7]. Hence (2) implies (3). ▮

The fact that (2) implies (3) in the above theorem was stated in [20]. The equivalence of (2) and (3) was shown in [18] for nesting over a single attribute.

THEOREM 7. *Assume $X \cap Z = \varnothing$ and $W = U - XZ$; thus the scheme of r is $U = XZW$. Then the following statements are equivalent*:

(1) *$X \twoheadrightarrow Z \mid W$ holds in r.*

(2) *$X \rightarrow Z^*$ holds in $r^* = \mathrm{NEST}_Z(r)$,*

(3) *$X \rightarrow W^*$ holds in $\mathrm{NEST}_W(r)$.*

(4) *$X \rightarrow Z^*W^*$ holds in $\mathrm{NEST}_W(\mathrm{NEST}_Z(r))$.*

(5) *$X \rightarrow Z^*W^*$ holds in $\mathrm{NEST}_Z(\mathrm{NEST}_W(r))$.*

Proof. Conditions (1) and (2) are equivalent by Theorem 6 as are (1) and (3). Again, we may apply Theorem 6 to conclude that (1) holds if and only if $X \twoheadrightarrow Z^*$ holds in $\mathrm{NEST}_Z(r)$. Note that the scheme of $\mathrm{NEST}_Z(r)$ is XZ^*W. By complementation we conclude (1) holds if and only if $X \twoheadrightarrow W$ holds in $r^* = \mathrm{NEST}_Z(r)$. Applying Theorem 6 to r^* with nesting on W, we conclude that $X \twoheadrightarrow W$ holds in $r^* = \mathrm{NEST}_Z(r)$ if and only if $X \rightarrow W^*$ holds in $\mathrm{NEST}_W(\mathrm{NEST}_Z(r))$, hence if and only if (1) holds. Furthermore, by Theorem 1, (2) holds if and only if $X \rightarrow Z^*$ holds in $\mathrm{NEST}_W(\mathrm{NEST}_Z(r))$, hence if and only if (1) holds. Thus, (1) is equivalent to (4) from the union rule for FDs. The fact that (1) is equivalent to (5) follows by a symmetric argument. ▮

TABLE II

	r'	
X	Z	W
x	z_1	w_1
x	z_2	w_2

We now show that the presence of a nontrivial MVD on r is sufficient to guarantee that certain NEST operators commute [18, 23, 15].

THEOREM 8. *Assume* $X \cap Z = \varnothing$, $W = U - XZ$. *If* $X \twoheadrightarrow Z \mid W$ *holds in* r *then* $\mathrm{NEST}_W(\mathrm{NEST}_Z(r)) = \mathrm{NEST}_Z(\mathrm{NEST}_W(r))$.

Proof. From Theorem 7, $X \to Z^* W^*$ holds in both $\mathrm{NEST}_W(\mathrm{NEST}_Z(r))$ and $\mathrm{NEST}_Z(\mathrm{NEST}_W(r))$. This means that for each X-value in $\Pi_X(r)$ there is only one tuple in $\mathrm{NEST}_W(\mathrm{NEST}_Z(r))$ with that X-value, i.e., if $\langle x, S, T\rangle$ and $\langle x, S', T'\rangle$ are tuples of $\mathrm{NEST}_W(\mathrm{NEST}_Z(r))$ then $S = S'$ and $T = T'$. Similarly, there is only one tuple in $\mathrm{NEST}_Z(\mathrm{NEST}_W(r))$ for each distinct X value. For a given x, let $\langle x, S_1, T_1\rangle$ denote the tuple in $\mathrm{NEST}_W(\mathrm{NEST}_Z(r))$ and $\langle x, S_2, T_2\rangle$ denote the corresponding tuple of $\mathrm{NEST}_Z(\mathrm{NEST}_W(r))$. Let $\sigma_{X=x}(r)$ be the subrelation of r having all tuples with X value x. Then $S_1 = \Pi_Z(\sigma_{X=x}(r)) = S_2$ and $T_1 = \Pi_W(\sigma_{X=x}(r)) = T_2$. This gives the desired equalities. ▌

Remark 2. The converse of Theorem 8 does not hold. A counterexample is given in Table II. The relation r' does not satisfy the MVD $X \twoheadrightarrow Z \mid W$. However, $\mathrm{NEST}_W(\mathrm{NEST}_Z(r')) = \mathrm{NEST}_Z(\mathrm{NEST}_W(r'))$ as can be verified by the reader.

Remark 3. There is no direct analogue of Theorem 3 for MVDs. In Table III, r satisfies $XZ \twoheadrightarrow Y$ but $r^* = \mathrm{NEST}_Z(r)$ violates $XZ^* \twoheadrightarrow Y$. Furthermore, the relation $\bar{r}^*$ consisting of the first and last tuples or r^* in Table III satisfies $XZ^* \twoheadrightarrow Y$, but $\bar{r} = \mathrm{UNNEST}_{Z^*}(\bar{r}^*)$ violates $XZ \twoheadrightarrow Y$.

TABLE III

	r				r^*		
X	Z	Y	W	X	Z^*	Y	W
x	z	y	w	x	$\{z, z'\}$	y	w
x	z	y'	w	x	$\{z\}$	y'	w
x	z	y	w'	x	$\{z\}$	y	w'
x	z	y'	w'	x	$\{z\}$	y'	w'
x	z'	y	w				

5. Local Dependencies

Our discussion now turns to the interaction between nesting and dependencies which are not necessarily of a global nature. A horizontal decomposition of a relation will be defined in terms of a partitioning of the set of tuples of r. Thus, a horizontal decomposition is a collection B of sets of tuples, called *blocks*, such that the blocks are pairwise disjoint and their union is all of the tuples of r.

Given any horizontal decomposition B of r, a dependency is said to be *local* to a block $b \in B$ if the dependency holds in b, where b is viewed as a separate relation. It is said to be *uniformly local* if it holds for all blocks $b \in B$. A dependency is *global* if it holds for all of r. Clearly, for the trivial decomposition B containing all of r all three notions are equivalent.

Any nesting operation induces a horizontal decomposition on r, where each block b of the partition B consists of those tuples of r which contribute to a tuple of r^*. Alternatively, given $r^* = \text{NEST}_Z(r)$, each block $b \in B$ can be obtained by performing $\text{UNNEST}_{Z^*}(\{t^*\})$, where t^* is a single tuple of r^*.

Just as nesting produces a particular FD even in the presence of no previous dependencies (Lemma 1 above), unnesting produces a certain MVD. We give the result for two nests; it easily generalizes to $n > 2$ nests over disjoint sets of attributes.

Lemma 3. *Let X, Z, W partition U. Let t^* be any tuple in* $\text{NEST}_W(\text{NEST}_Z(r))$. *Then the* MVD $X \twoheadrightarrow Z \mid W$ *holds in* $\text{UNNEST}_{Z^*}(\text{UNNEST}_{W^*}(\{t^*\}))$.

Proof. t^* is of the form $\langle x, S, T\rangle$. The block induced by unnesting will contain tuples of the form $\langle x, z, w\rangle$ for all $\langle z, w\rangle \in S \times T$. ∎

We may interpret Lemma 3 as saying the MVD will hold uniformly locally in the horizontal decomposition induced by $\text{NEST}_W(\text{NEST}_Z(r))$.

In general, global FDs will induce uniformly local FDs. Hence one seeks results where the presence of certain local FDs will guarantee a global FD. We believe the following theorem is a good characterization of this property. Again, $r^* = \text{NEST}_Z(r)$.

Theorem 9. *Let $W \cap Z = \varnothing$ and $XY \subset Z$. Then $W \twoheadrightarrow Z$ and $WX \rightarrow Y$ hold globally in r if and only if $W \rightarrow Z^*$ holds globally in r^* and $X \rightarrow Y$ holds uniformly locally in r^*.*

Proof. If $W \twoheadrightarrow Z$ holds in r, then $W \rightarrow Z^*$ holds in r^* from Theorem 6. Since $W \cap Z = \varnothing$, all tuples in a block induced by r^* must have the same W value. Since $WX \rightarrow Y$ holds globally in r, $X \rightarrow Y$ holds within each block, i.e., holds uniformly locally in r^*.

Now suppose $W \rightarrow Z^*$ holds globally in r^* and $X \rightarrow Y$ holds locally. Then $W \twoheadrightarrow Z$ holds in r by Theorem 6. Now consider tuples of the form $t_1 = \langle w, x, y, s, v\rangle$ and $t_2 = \langle w, x, y', s', v'\rangle$ in r, where s, $s' \in Z - XY$ and v, $v' \in U - WZ$. To show that $WX \rightarrow Y$ holds in r we need to show $y = y'$. Since

$W \twoheadrightarrow Z$ holds, the tuple $t_3 = \langle w, x, y', s', v \rangle$ is also in r. But t_1 and t_3 are in the same block of the decomposition induced by r^*, since they agree on $U - Z$. Hence the desired equality $y = y'$ follows from the local dependency $X \to Y$. ▮

COROLLARY 4. *Let $XY \subset Z$. Then $(U-Z)X \to Y$ holds in r if and only if $X \to Y$ holds uniformly locally in r^*.*

Proof. The MVD $(U-Z) \twoheadrightarrow Z$ holds trivially in r and the FD $(U-Z) \to Z^*$ holds in r^* by Lemma 1. The result follows immediately from Theorem 9. ▮

A result analogous to Theorem 9 holds for MVDs.

THEOREM 10. *Let $W \cap Z = \varnothing$ and $XY \subset Z$. Then $W \twoheadrightarrow Z$ and $WX \twoheadrightarrow Y$ hold in r if and only if $W \to Z^*$ holds globally in r^* and $X \twoheadrightarrow Y$ holds uniformly locally in r^*.*

Proof. As before $W \twoheadrightarrow Z$ holds in r if and only if $W \to Z^*$ holds in r^* from Therem 6. Consider tuples of the form $\langle x, x, y, s, v \rangle$ and $\langle w, x, y', s', v \rangle$ in the same block b of the decomposition induced by r^* with s, $s' \in Z - XY$ and $v \in Y - WZ$. From the MVD $WX \twoheadrightarrow Y$ we conclude that $\langle w, x, y', s, v \rangle$ is in r and in the same block b. Thus, the block b satisfies $X \twoheadrightarrow Y$. For the converse, consider the tuples t_1 and t_2 in the proof of Theorem 9. We know $t_3 \in r$ and we wish to show $t_4 = \langle w, x, y', s, r \rangle \in r$. Since t_1 and t_3 are in the same block of r^* the local dependency $X \twoheadrightarrow Y$ will yield the desired result. ▮

COROLLARY 5. *Let $XY \subset Z$. Then $(U-Z)X \twoheadrightarrow Y$ holds in r if and only if $X \twoheadrightarrow Y$ holds uniformly locally in r^*.*

Proof. Immediate from Theorem 10. ▮

6. DISCUSSION

In Sections 3 and 4 of this paper, we studied how dependencies (FDs and MVDs) interact with nesting. The following cases gave pleasing results:

(1) nesting away from a dependency;

(2) nesting on the right-hand side of a dependency;

(3) nesting on a part of the right-hand side of an FD;

(4) nesting on a part of the left-hand side of an FD.

When nesting is done away from a dependency holding in r the same dependency holds in the nested relation r^* (Theorems 1 and 5). Since r^* has less redundancy than r, dependency checking can be done more efficiently. Although we only deal with FDs and MVDs we believe that this result can be generalized to a larger class of dependencies.

When nesting is done on the right-hand side or part of the right-hand side of an FD (Lemma 2 and Theorem 2), singleton sets are produced in the Z^* column of the nested relation, thus essentially creating the same relation. We therefore believe that nesting in this case should be avoided.

Theorem 4 gives a result for nesting on part of the left-hand side of an FD. Basically, it says that to enforce the original FD $X \to Y$, rewritten as $(X-Z)Z^* \to Y$, in nested form, we replace the condition that two Z values be equal by the condition that two sets over Z^* overlap (i.e., have a nonempty inersection). We also showed that a similar result for MVDs does not hold.

The situation in Theorem 3 does not fit neatly into the above characterization. The essence of that theorem is that the left-hand side of the FD determines $U-Z$, i.e., all of the unnested attributes.

The most interesting results are obtained when nesting is performed on the right-hand side of an MVD $X \twoheadrightarrow Z$. Theorem 6 gives an alternative characterization of MVDs, independent of any notion of vertical decomposition, i.e., the MVD $X \twoheadrightarrow Y$ holds in r if and only if the FD $X \to Z^*$ holds in $r^* = \text{NEST}_Z(r)$. Moreover, this characterization is quite natural as the following classical example illustrates. Suppose we want to maintain a database of employees (E), the children (C) of each employee and the departments (D) each employee works for. It is well known that the MVD $E \twoheadrightarrow C|D$ must hold. The intuitive meaning of this dependency is that an employee has a unique set of children and a unique set of departments he or she works for. It is therefore natural that the dependency $E \to C^*D^*$ holds in $r^{**} = \text{NEST}_C(\text{NEST}_D(r))$. (We also know $r^{**} = \text{NEST}_D(\text{NEST}_C(r))$ by Theorem 8.) The interesting and important side effect is that r has been converted into a representation satisfying 4NF without vertical decomposition (cf. [13]). Furthermore since $\text{UNNEST}_C(\text{UNNEST}_D(r^{**})) = r$, we have not lost any information. We believe that this result could be a useful contribution to normalization theory and the role of MVDs in relational database design.

Since nesting induces a horizontal decomposition of r we studied how global dependencies holding in r induce similar but "stronger" dependencies in the blocks of r and vice versa. The fact that each block also satisfies certain stronger dependencies might lead to some efficiencies in query processing. Theorems 9 and 10 essentially tell us that in the presence of an MVD $X \twoheadrightarrow Z$, dependencies "embedded" in the right-hand side Z are not affected by nesting on Z.

Acknowledgment

The authors gratefully acknowledge the helpful comments and suggestions of the referee.

References

1. W. W. Armstrong, Dependency structures of data base relationships, *Inform. Process.* **74** (1974), 580–583.

2. S. ABITEBOUL AND N. BIDOIT, Non first normal form relations to represent hierarchically organized data *in* "Proc. Third ACM SIGACT-SIGMOD Symposium on Principles of Database Systems," 1984, pp. 191–200.
3. W. W. ARMSTRONG AND C. DELOBEL, Decomposition and functional dependencies in relations, *ACM Trans. Database Systems* **5**, No. 4 (1980), 404–430.
4. T. BANCILHON, P. RICHARD, AND M. SCHOLL, On line processing of compacted relations *in* "Proc. Eighth Internat. Conf. on Very Large Data Bases," 1982, pp. 263–269.
5. C. BEERI AND P. A. BERNSTEIN, Computational problems related to the design of normal form relational schemes, *ACM Trans. Database Systems* **4**, No. 1 (1979), 30–59.
6. C. BEERI, P. A. BERNSTEIN, AND N. GOODMAN, A sophisticate's introduction to database normalization theory, *in* "Proc. Fourth Internat. Conf. on Very Large Databases," 1978, pp. 113–124.
7. C. BEERI, R. FAGIN, AND J. H. HOWARD, A complete axiomatization for functional and multivalued dependencies in database relations, *in* "Proc. ACM SIGMOD Internat. Conf. on Management of Data, 1977, 47–61.
8. P. A. BERNSTEIN, Synthesizing third normal form relations from functional dependencies, *ACM Trans. Database Systems* **1**, No. 4 (1976), 277–298.
9. P. A. BERNSTEIN, J. R. SWENSON, AND D. C. TSICHRITZIS, A unified approach to functional dependencies and relations, *in* "Proc. ACM SIGMOD Internat. Conf. on Management of Data," 1975, pp. 237–245.
10. E. F. CODD, A relational model for large shared data banks, *Comm. ACM* **13**, No. 6 (1970), 377–387.
11. E. F. CODD, Relational completeness of database sublanguages, "Database Systems" (R. Rustin, Ed.), Prentice–Hall, Englewood Cliffs, N. J., 1972.
12. C. DELOBEL, Normalization and hierarchical dependencies in the relational data model, *ACM Trans. Database Systems* **3**, No. 3 (1978), 201–222.
13. R. FAGIN, Multivalued dependencies and a new normal form for relational databases, *ACM Trans. Database Systems* **2**, No. 3. (1977), 262–278.
14. P. C. FISCHER AND S. J. THOMAS, Operators for non-first-normal form relations, *in* "Proc. IEEE Computer Software and Applications Conference," 1983, pp. 464–475.
15. P. C. FISCHER AND D. VAN GUCHT, Weak multivalued dependencies, *in* "Proc. Third ACM SIGACT-SIGMOD Symposium on Principles of Database Systems," 1984, pp. 266–274.
16. A. FURTADO AND L. KERSCHBERG, An algebra of quotient relations, *in* "Proc. ACM SIGMOD Internat Conf. on Management of Data," 1977, pp. 1–8.
17. G. GONNET, Unstructured data bases, *in* "Proc. Second ACM SIGACT-SIGMOD Symposium on Principles of Database Systems," 1983, pp. 117–124.
18. G. JAESCHKE AND H.-J. SCHEK, Remarks on the algebra on non first normal form relations *in* "Proc. ACM SIGACT-SIGMOD Symposium on Principles of Database Systems," 1982, pp. 124–138.
19. D. LUO AND S. B. YAO, Form operation by example, *in* "Proc. ACM SIGMOD Internat. Conf. on Management of Data," 1981, pp. 212–223.
20. A. MAKINOUCHI, A consideration of normal form of not-necessarily-normalized relations in the relational data model, *in* "Proc. Fifth Internat. Conf. on Very Large Data Bases," 1977, pp. 447–453.
21. Z. M. OZSOYOGLU AND G. OZSOYOGLU, An extension of relational algebra for summary tables, *in* "Proc. Statistical Database Workshop," 1983, pp. 202–211.
22. P. DE BRA AND J. PAREDAENS, An algorithm for horizontal decomposition, *Inform. Process. Lett.* **17**, No. 2 (1983), 91–95.
23. S. J. THOMAS, "A Non-First-Normal Form Relational Database Model," Ph. D. dissertation, Vanderbilt University, 1983.
24. S. J. THOMAS AND P. C. FISCHER, Nested relational structures, *in* "The Theory of Databases" (P. C. Kanellakis, Ed.), JAI Press, Greenwich, Conn., in press.
25. C. ZANIOLO, "Analysis and Design of Relational Schemata for Database Systems," Ph. D. dissertation, UCLA, 1976.